Thermomechanical noise in micromachined sensors

The fundamental specification for MEMS test equipment?

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Outline

- Introduction
- Noise theory
- Experimental validation
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- Case II: MEMS flow meter resolution
- Conclusion
Noise theory

The fundamental noise limit for micromechanical flow sensors is given by eqn. (14) with currently the best results of de Graaf et al. [9]. This sensor was made of a channel with a diameter of approximately 14 μm and a length of 1 m. It was reported that the dominant noise is caused by electronics.

The noise theory has been studied for micromechanical systems [10], atomic force microscopy [11], and beams [17]. However, the noise at the fundamental noise limit has never been studied. Here, we validate the result by measuring the mechanical noise on the channel of a micromechanical flow sensor.

The torque on the channel is defined as

\[ \tau = \sqrt{4 k_B T R}, \]

which can be interpreted as a mechanical Johnson–Nyquist noise. In our experiments, we measured the displacement spectral density \( |x_n(\omega)| \) at the channels 1 and 2 in Fig. 1. Substituting Eq. (14) into Eq. (9) and performing a limit by \( L/2 \) gives an expression for this noise density as a function of frequency:

\[ |x_n(\omega)| \approx \frac{L}{2} \sqrt{\frac{k_B T}{m L^2} \omega}. \]
Modeling noise

- Thermomechanical noise.
- Johnson-Nyquist noise.
- Electrical engineers ❤

lumped element modeling!
A simple system

- ‘Kinetic’ energy (inductor)
  \[ E_k = \frac{1}{2} Li^2 \]

- ‘Potential’ energy (capacitor)
  \[ E_p = \frac{1}{2} Cv^2 \]
A simple system, but WRONG!

• ‘Lost’ energy in the resistor
  \[ P_R = \frac{E}{t} = i^2 R \]

• Ideal electrical system. Infinitesimal heat capacity.

• Conservation of energy?
Equipartition theorem

\[ \sum_{\text{storage}} \langle E \rangle = k_B T \]

A simple system

- Equipartition theorem
  \[ \sum_{\text{storage}} \langle E \rangle = k_B T \]

- Energy storage in this system
  \[ \frac{1}{2} L \langle i_n^2 \rangle + \frac{1}{2} C \langle v_n^2 \rangle = k_B T \]
A simple system

- Equipartition theorem
  \[ \sum_{\text{storage}} \langle E \rangle = k_B T \]

- Energy storage in this system
  \[ \frac{1}{2} L \langle i_n^2 \rangle + \frac{1}{2} C \langle v_n^2 \rangle = k_B T \]
A simple system

- Apply some Kirchhoff and Fourier...
  \[ i_n(\omega) = \frac{v_n(\omega)}{j\omega RC - \omega^2 LC + 1} \]
- Calculate the average...
  \[ \langle i_n^2 \rangle = \frac{1}{2\pi} \int_0^\infty i_n(\omega)^2 d\omega \]
- Apply equipartition theorem...
  \[ L\langle i_n^2 \rangle = k_B T \]
- Result....
  \[ v_n^2 = 4k_B TR \]
Johnson–Nyquist noise

\[ \nu_n = \sqrt{4k_B T R} \]
Johnson–Nyquist noise

- ‘Kinetic’ energy (mass)
  \[ E_k = \frac{1}{2} Mu^2 \]

- ‘Potential’ energy (spring)
  \[ E_p = \frac{1}{2} Kx^2 \]
Johnson–Nyquist noise

\[ F_n = \sqrt{4k_B T D} \]
Some practical tips on this trick...

• Take care of multiple degrees of freedom.

• Think outside the boundaries of one domain.

• Make sure all ‘energy storages’ of the system are taken into account.
Experimental validation

The fundamental limit for continuous flow sensors is given by currently the best results of J. [9]. This sensor was a channel with a diameter of approximately 14 nm s^-1 was reported. The dominant noise is caused by electronics.

The fundamental noise limit has not been studied for micro \[ J \] atomic force microscopy yet. However, the noise has never been studied. Here, we validate the result by mea-sure noise equivalent mass flow. The noise equivalent noise on the thermomechanical noise on the defining the noise equivalent flow sensors.

\[ J = \sqrt{\frac{4k_B T \omega Q}{L^2}} \]

with \( m \) the mass of the channel with the fluid. The noise angle density squared \( \langle \theta_n^2 \rangle \) follows from:

\[ \langle \theta_n^2 \rangle = \frac{1}{2\pi} \int_0^\infty |\theta_n(\omega)|^2 d\omega. \]

And thus, Eq. (9) can be solved.

\[ \frac{K}{2\pi} \int_0^\infty |\theta_n(\omega)|^2 d\omega = k_B T. \]

Inserting Eq. (12) into Eq. (9) and performing gives the noise torque spectral density \( \tau_n \):

\[ \tau_n = \sqrt{4k_B T R}, \]

which can be interpreted as a mechanical Johnson–Nyquist noise. In our experiments, we measured equilibrium density \( |x_n(\omega)| \) at the channels 1 and 2 in Fig. 1. Substituting Eq. (14) into Eq. (9) and by \( L/2 \) gives an expression for this noise density as a function of frequency:

\[ |x_n(\omega)| \approx \frac{L}{2} \sqrt{4k_B T \omega Q}. \]
Measurement setup

Measurement setup

Model fits

- Second order mechanical system.
- Johnson-Nyquist noise.

\[ x_n = \frac{L \sqrt{4k_B T \frac{\omega_0 J}{Q}}}{2J\omega_0 \sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \frac{1}{Q^2} \left(\frac{\omega}{\omega_0}\right)^2}} \]
Experimental validation

- RMS for a bandwidth of 13 Hz around the resonance frequency.
- The thermomechanical noise increases with temperature.
- Decent values for other fitting parameters (i.e. $\omega_0$, $J$ and $Q$).
Case I: MEMS accelerometer resolution

The fundamental limit to the resolution of flow sensors is given by the dominant noise. The dominant noise is caused by electronics.

The fundamental noise limit on the resolution of flow sensors is given by the electronic noise. The electronic noise is caused by the current in the channel with a diameter of approximately 14 ng s^{-1} was reported. The noise is primarily caused by the vibrations of the channel.

The noise angle density squared \( \langle \theta_n^2 \rangle \) follows from the electronic noise:

\[ \langle \theta_n^2 \rangle = \frac{1}{2\pi} \int_0^\infty |\theta_n(\omega)|^2 d\omega. \]

And thus, Eq. (9) can be solved:

\[ \frac{K}{2\pi} \int_0^\infty |\theta_n(\omega)|^2 d\omega = k_BT. \]

Inserting Eq. (12) into Eq. (9) and performing the integral gives the noise torque spectral density \( \tau_n \):

\[ \tau_n = \sqrt{4k_BT}, \]

which can be interpreted as a mechanism for Johnson–Nyquist noise. In our experiments, we observed the displacement spectral density \( |x_n(\omega)| \) at the channel 1 and 2 in Fig. 1. Substituting Eq. (14) into Eq. (12) gives an expression for the noise density as a function of frequency:

\[ |x_n(\omega)| \approx \frac{L}{2} \sqrt{4k_BT \frac{\omega^2}{\theta Q}}. \]
Accelerometer

\[ \Delta y \propto a \]
Accelerometer

- Noise acceleration
  \[ a_n = 240 \, \mu g/\sqrt{\text{Hz}} \]

- Displacement
  \[ x_n = 5 \, \text{pm}/\sqrt{\text{Hz}} \]

- Difference in capacitance
  \[ C_n = 600 \, \text{zF}/\sqrt{\text{Hz}} \]
Case II: MEMS flow meter resolution
Microfabricated Coriolis flow sensor
Microfabricated Coriolis flow sensor
Microfabricated Coriolis flow sensor

\[ F_c \propto \Phi \]
Microfabricated Coriolis flow sensor

- Twist noise on twist.
- Twist noise on swing.
- Swing noise on swing.
- Swing noise on twist.
Microfabricated Coriolis flow sensor

• Limited by noise when
  \[ F_c = F_{\text{swingnoise@twistfrequency}} \]

• Mass flow
  \[ \Phi_n = 0.3 \text{ ng s}^{-1}/\sqrt{\text{Hz}} \]

• Difference in phase
  \[ \phi_n = 3 \text{ n}^\circ/\sqrt{\text{Hz}} \]
Conclusion

The fundamental limit for continuous flow sensors is given by the noise angle density squared \( \langle \theta^2_n \rangle \) follows from

\[
\langle \theta^2_n \rangle = \frac{1}{2\pi} \int_0^\infty |\theta_n(\omega)|^2 d\omega.
\]

And thus, Eq. (9) can be solved.

\[
\frac{K}{2\pi} \int_0^\infty |\theta_n(\omega)|^2 d\omega = k_B T.
\]

Inserting Eq. (12) into Eq. (9) and performing a saddle point gives the noise torque spectral density \( \tau_n \):

\[
\tau_n = \sqrt{4k_B T R},
\]

which can be interpreted as a mechanism for Johnson–Nyquist noise. In our experiments, we found the displacement spectral density \( |x_n(\omega)| \) at the channel 1 and 2 in Fig. 1. Substituting Eq. (14) into Eq. (12) and solving by \( L/2 \) gives an expression for this noise density as a function of frequency:

\[
|x_n(\omega)| \approx \frac{L}{2} \sqrt{4k_B T \frac{\nu}{Q}}.
\]
Conclusion

• The Johnson-Nyquist noise model is a convenient method for multidomain analog resolution analysis.

• From two types of MEMS sensors, future specifications for capacitance meters and phase detectors are derived.

• These are in the order of aF and n° respectively.
Outlook

• In practice: sampling rate and high-voltage compatibility are more relevant at the moment.
Acknowledgements
