

# Thermomechanical noise in micromachined sensors

*The fundamental specification for MEMS test equipment?*

Dennis Alveringh

$$J = \frac{1}{7} mL^2,$$

with  $m$  the mass of the channel with the fluid, the noise angle density squared  $\langle \theta_n^2 \rangle$  follows from

$$\langle \theta_n^2 \rangle = \frac{1}{2\pi} \int_0^\infty |\theta_n(\omega)|^2 d\omega.$$

And thus, Eq. (9) can be solved.

$$K \int_0^\infty |\theta_n(\omega)|^2 d\omega = k_B T.$$

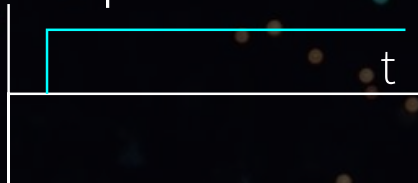
Inserting Eq. (12) into Eq. (9) and performing the integration gives the noise torque spectral density  $\tau_n$ :

$$\tau_n = \sqrt{4k_B T R},$$

which can be interpreted as a mechanical Johnson–Nyquist noise. In our experiments, we measured the displacement spectral density  $|x_n(\omega)|$  at the channels 1 and 2 in Fig. 1. Substituting Eq. (14) into Eq. (10) and multiplying by  $L/2$  gives an expression for this noise density as a function of frequency:

$$\sqrt{4k_B T \frac{\omega d}{Q}}$$

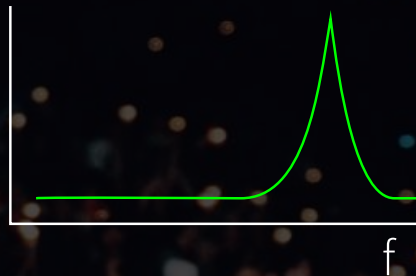
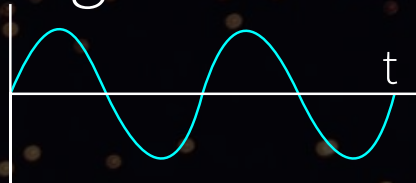
clap



low voice



high voice





# Outline

- Introduction
- Noise theory
- Experimental validation
- Case I: MEMS accelerometer resolution
- Case II: MEMS flow meter resolution
- Conclusion

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$$|x_n(\omega)| \approx \frac{L}{2} \cdot \frac{\sqrt{4k_B T \frac{\omega Q}{Q}}}{\sqrt{1 + \left(\frac{\omega Q}{\omega_0}\right)^2}}.$$

# Noise theory

The fundamental limit to the angular flow sensors is given by the thermal noise in the channel with currently the best resolution [9]. This sensor was able to measure a flow rate of approximately  $14 \text{ ng s}^{-1}$  was reported. The dominant noise is caused by the thermal noise in the channel.

Microfluidic channels have been studied for microfluidics [12], atomic force microscopy [13], and optical tweezers [14]. However, the noise in microfluidic channels has never been studied. Here, we study the fundamental noise limit in microfluidic channels, validate the result by measuring the noise equivalent mass flow, and compare it to the thermomechanical noise on the channel walls. This defines the noise equivalent mass flow for microfluidic flow sensors.

The channel is modeled as a second order system. The time constant of the channel is  $\tau_c$  and the resonance frequency is  $\omega_0$ .

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$$\tau_c = \frac{2L}{v} = \sqrt{J}, \quad J = \frac{1}{7} mL^2,$$

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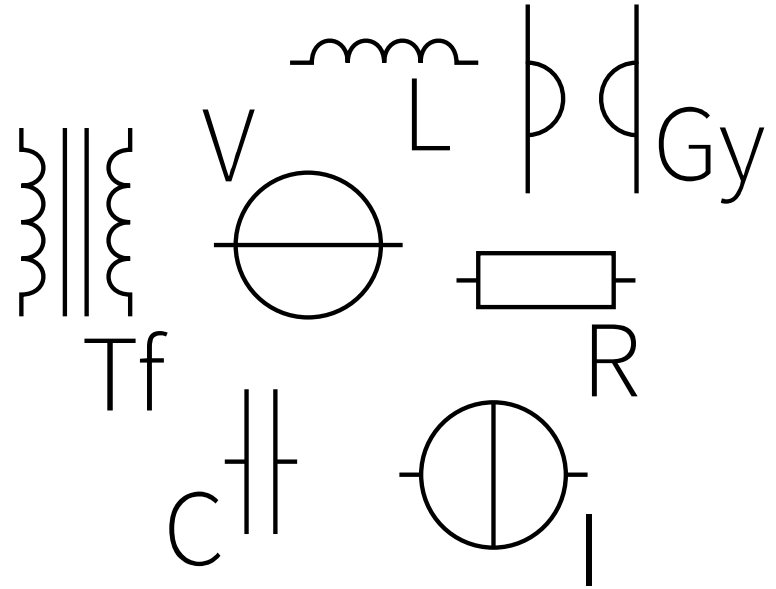
$$|x_n(\omega)| \approx \frac{L}{2} \cdot \frac{\sqrt{4k_B T \frac{\omega_0 L}{Q}}}{\sqrt{(\omega_0^2 - \omega^2)^2 + \frac{\omega^2}{Q^2}}}$$

# Modeling noise

- Thermomechanical noise.
- Johnson-Nyquist noise.
- Electrical engineers



lumped element  
modeling!



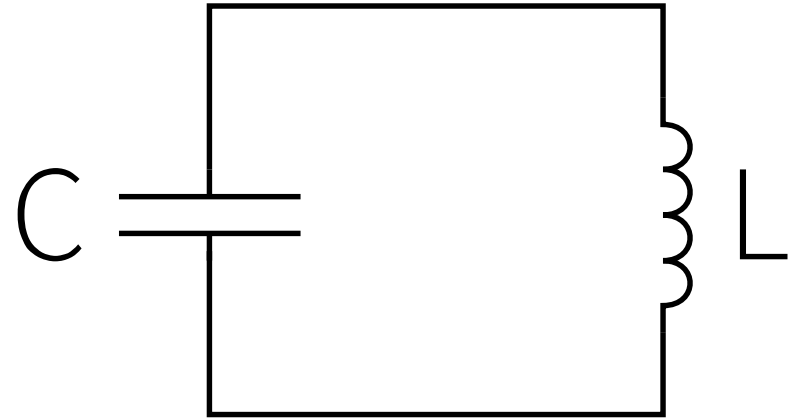
# A simple system

- ‘Kinetic’ energy (inductor)

$$E_k = \frac{1}{2} Li^2$$

- ‘Potential’ energy (capacitor)

$$E_p = \frac{1}{2} Cv^2$$

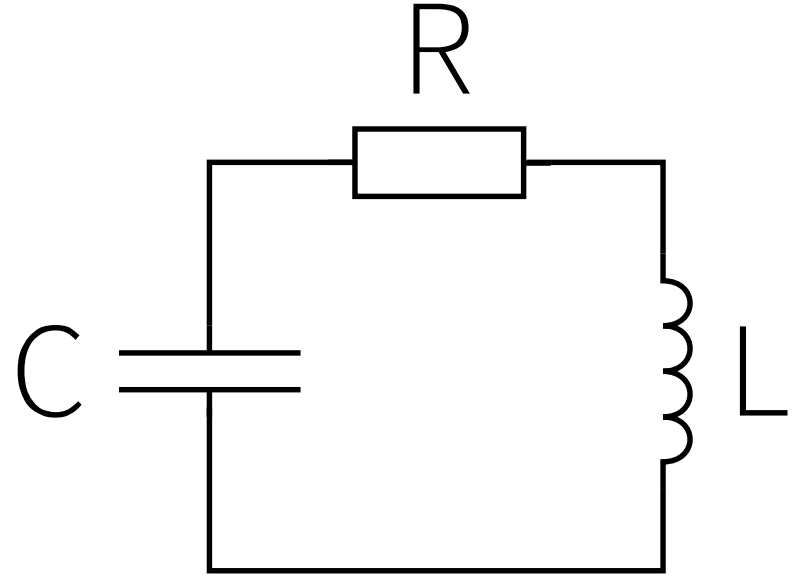


# A simple system, but **WRONG!**

- ‘Lost’ energy in the resistor

$$P_R = \frac{E}{t} = i^2 R$$

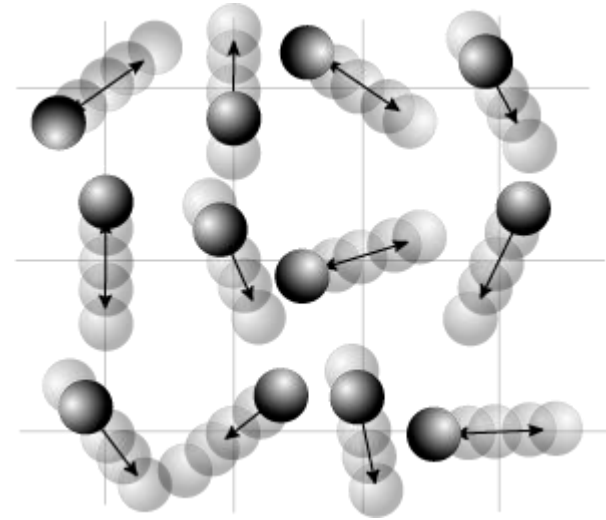
- Ideal electrical system.  
Infinitesimal heat capacity.
- Conservation of energy?






# Equipartition theorem

$$\sum_{\text{storage}} \langle E \rangle = k_B T$$



 D. Alveringh, *Experimental analysis of thermomechanical noise in micro Coriolis mass flow sensors*, Sensors and Actuators A: Physical 271: 212-216, 2018.

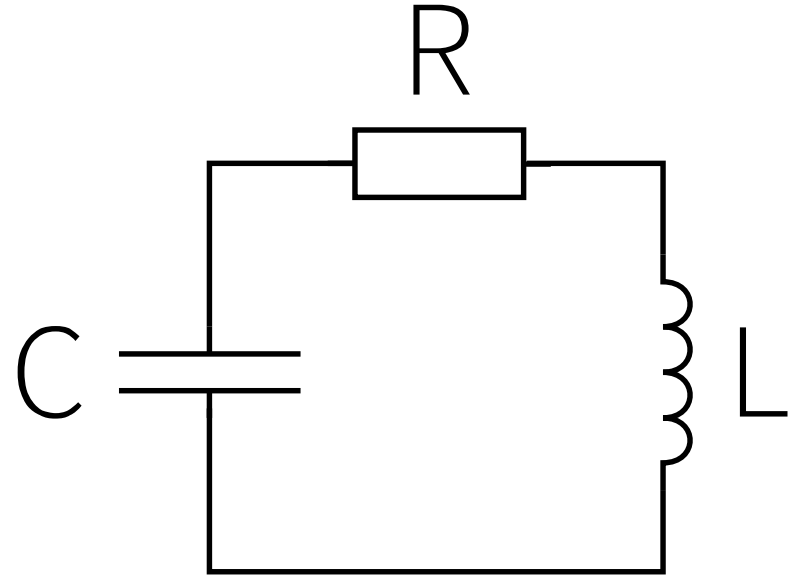
# A simple system

- Equipartition theorem

$$\sum_{\text{storage}} \langle E \rangle = k_B T$$

- Energy storage in this system

$$\frac{1}{2} L \langle i_n^2 \rangle + \frac{1}{2} C \langle v_n^2 \rangle = k_B T$$



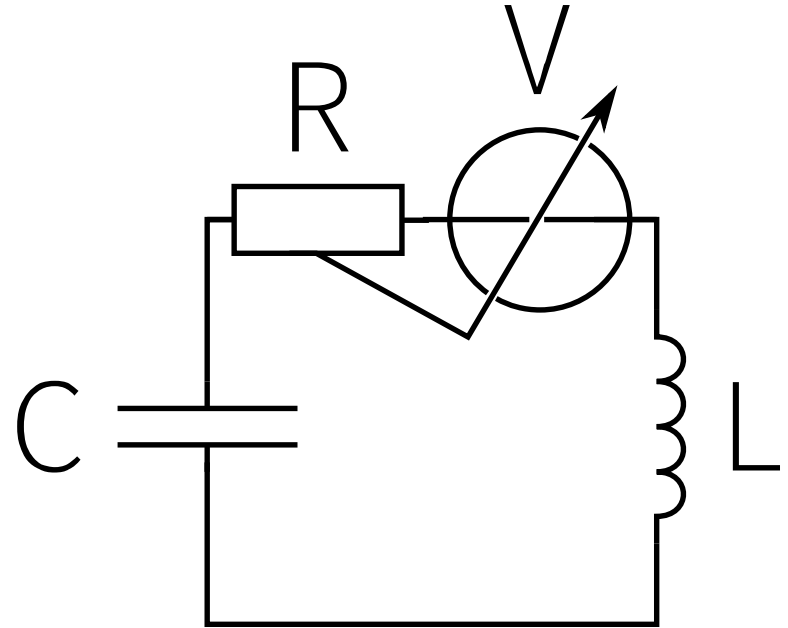
# A simple system

- Equipartition theorem

$$\sum_{\text{storage}} \langle E \rangle = k_B T$$

- Energy storage in this system

$$\frac{1}{2} L \langle i_n^2 \rangle + \frac{1}{2} C \langle v_n^2 \rangle = k_B T$$



## A simple system

- Apply some Kirchhoff and Fourier...

$$i_n(\omega) = \frac{v_n(\omega)}{j\omega RC - \omega^2 LC + 1}$$

- Calculate the average...

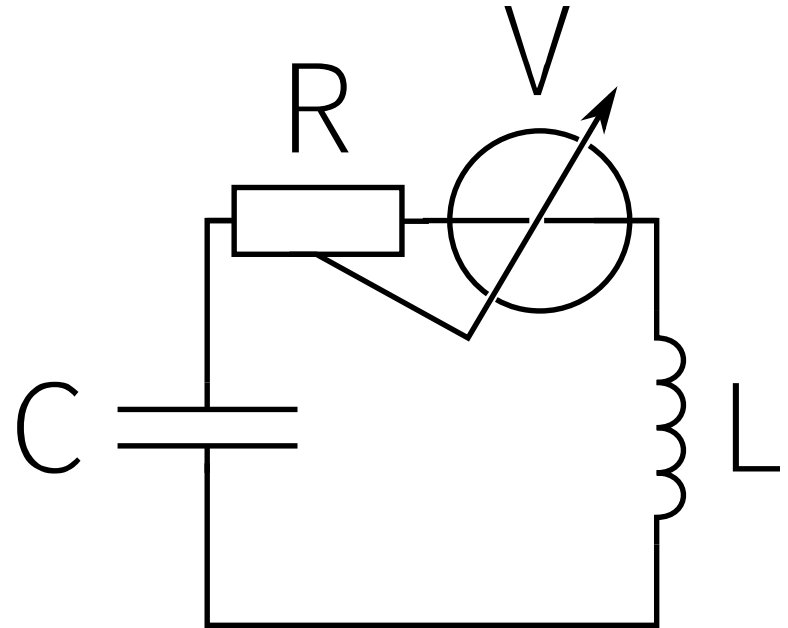
$$\langle i_n^2 \rangle = \frac{1}{2\pi} \int_0^\infty i_n(\omega)^2 d\omega$$

- Apply equipartition theorem...

$$L \langle i_n^2 \rangle = k_B T$$

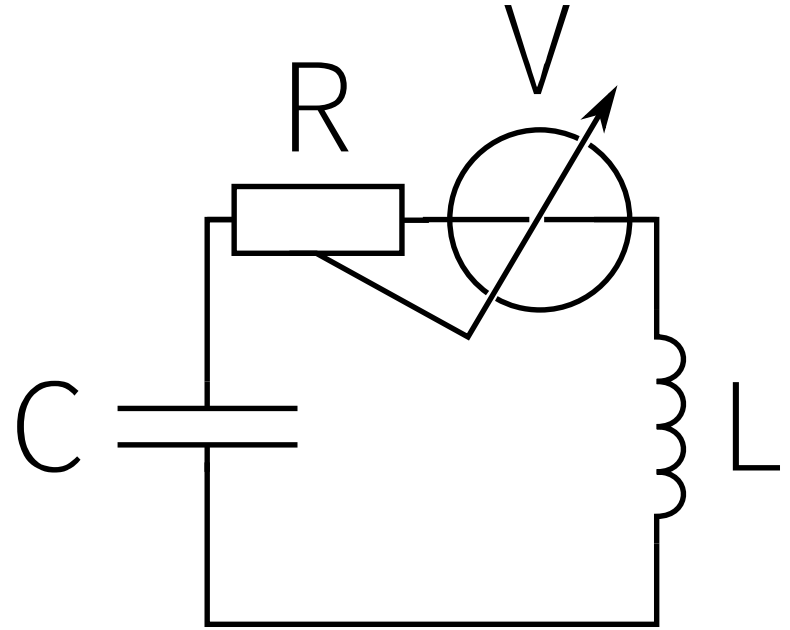
- Result....

$$v_n^2 = 4k_B T R$$



# Johnson–Nyquist noise

$$v_n = \sqrt{4k_B T R}$$



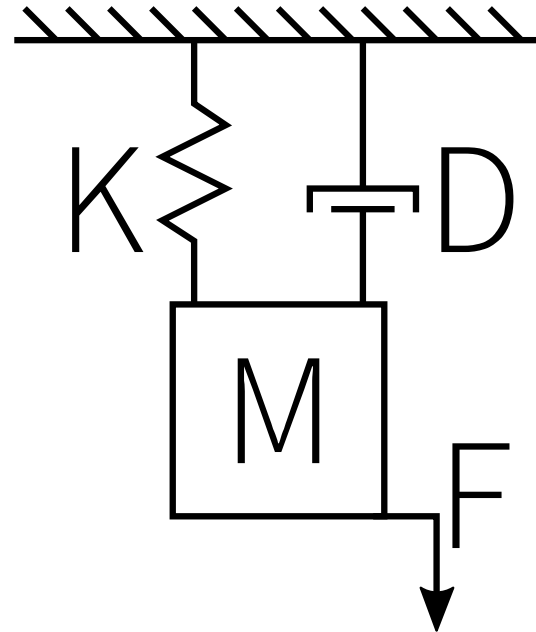
# Johnson–Nyquist noise

- ‘Kinetic’ energy (mass)

$$E_k = \frac{1}{2} M u^2$$

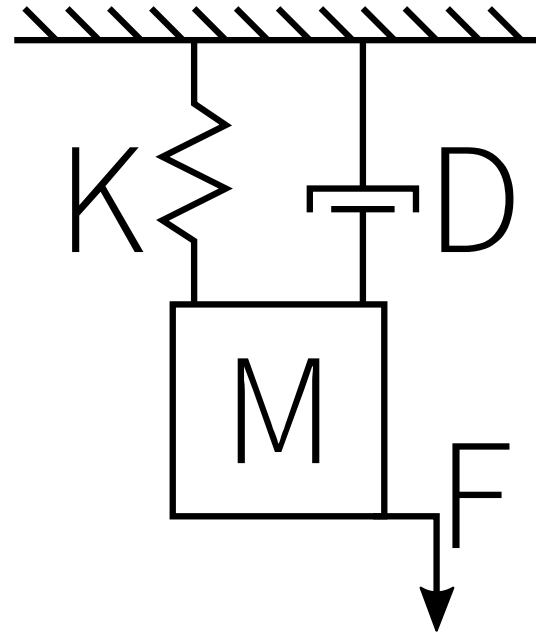
- ‘Potential’ energy (spring)

$$E_p = \frac{1}{2} K x^2$$



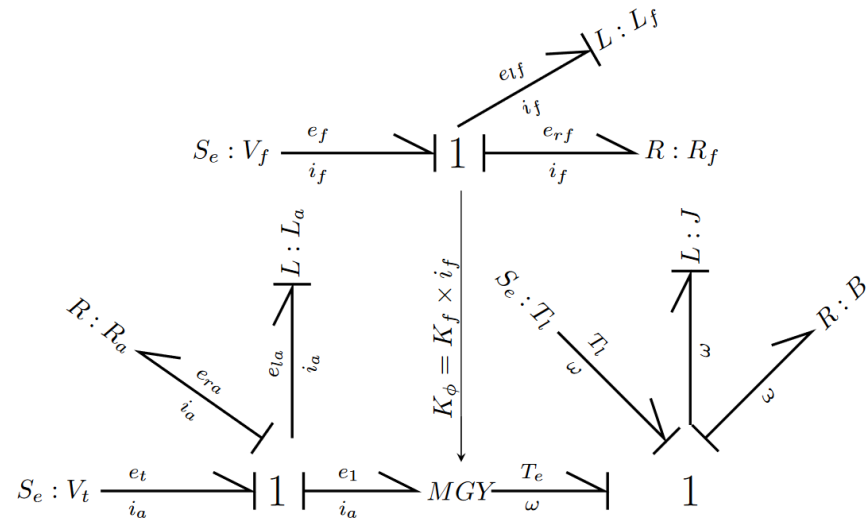
# Johnson–Nyquist noise

$$F_n = \sqrt{4k_B T D}$$



## Some practical tips on this trick...

- Take care of multiple degrees of freedom.
- Think outside the boundaries of one domain.
- Make sure all ‘energy storages’ of the system are taken into account.





# Experimental validation

The fundamental limit to the minimum flow sensors is given by the noise floor with currently the best resolution [9]. This sensor was used in a channel with a diameter of  $100\ \mu\text{m}$  and a flow rate of approximately  $14\ \text{ng s}^{-1}$  was reported. The dominant noise is caused by electronics.

Microfluidic channels have been studied for microfluidic sensors such as atomic force microscopy (AFM) [10]. In this study, the noise floor was studied. Here, we study the noise floor, the fundamental noise limit, and validate the result by measuring the noise equivalent mass flow. We compare thermomechanical noise on the channel with the noise equivalent mass flow sensors.

The channel can be modeled as a second order system. The time constant is  $\tau = \frac{m}{k}$  and the quality factor is  $Q = \frac{m\omega_0}{k}$ .

$$\tau = \frac{m}{k} = \sqrt{J}, \quad J = \frac{1}{7}mL^2,$$

with  $m$  the mass of the channel with the fluid. The noise angle density squared  $\langle \theta_n^2 \rangle$  follows from

$$\langle \theta_n^2 \rangle = \frac{1}{2\pi} \int_0^\infty |\theta_n(\omega)|^2 d\omega.$$

And thus, Eq. (9) can be solved.

$$\frac{K}{2\pi} \int_0^\infty |\theta_n(\omega)|^2 d\omega = k_B T.$$

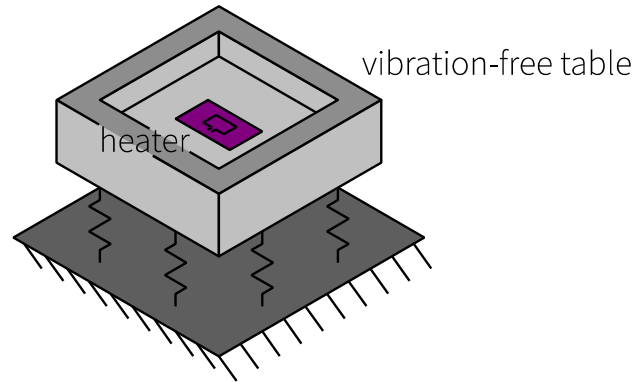
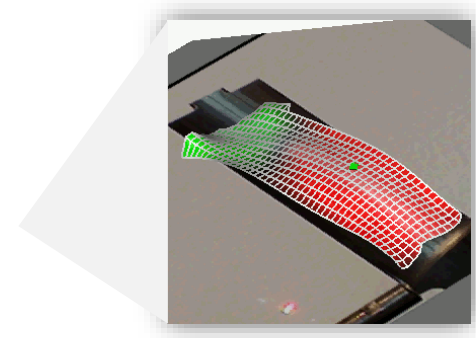
Inserting Eq. (12) into Eq. (9) and performing the integration gives the noise torque spectral density  $\tau_n$ :

$$\tau_n = \sqrt{4k_B T R},$$

which can be interpreted as a mechanical Johnson–Nyquist noise. In our experiments, we measure the displacement spectral density  $|x_n(\omega)|$  at the channel. The noise torque spectral density  $\tau_n$  is related to the displacement spectral density  $|x_n(\omega)|$  by  $L/2$  giving an expression for this noise density as a function of frequency:

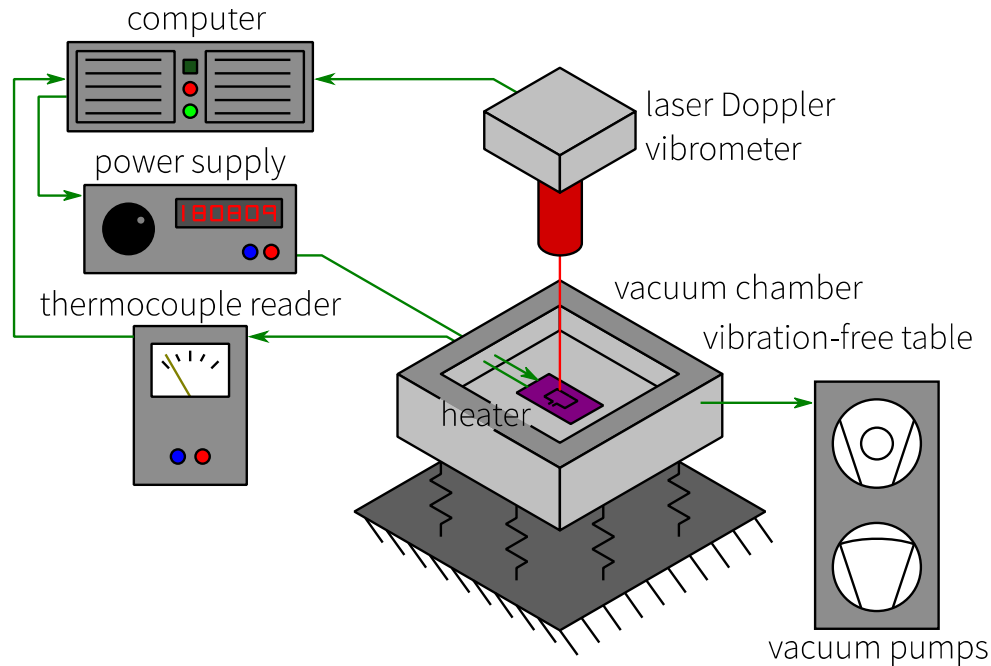
$$|x_n(\omega)| \approx \frac{L}{2} \cdot \frac{\sqrt{4k_B T \frac{\omega_0}{Q}}}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

# Measurement setup



 D. Alvering, *Integrated throughflow mechanical microfluidic sensors*, PhD dissertation, 2018.

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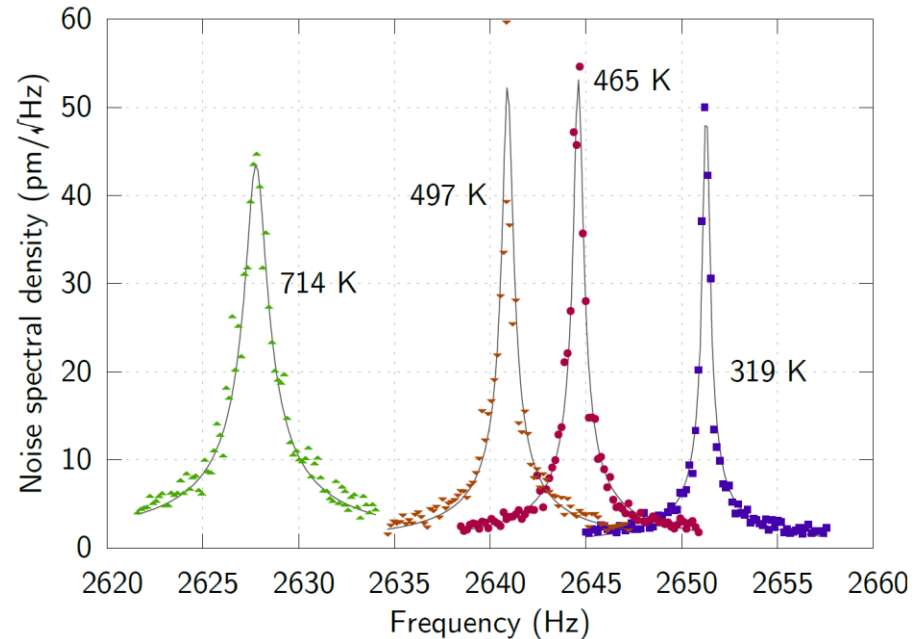


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# Model fits

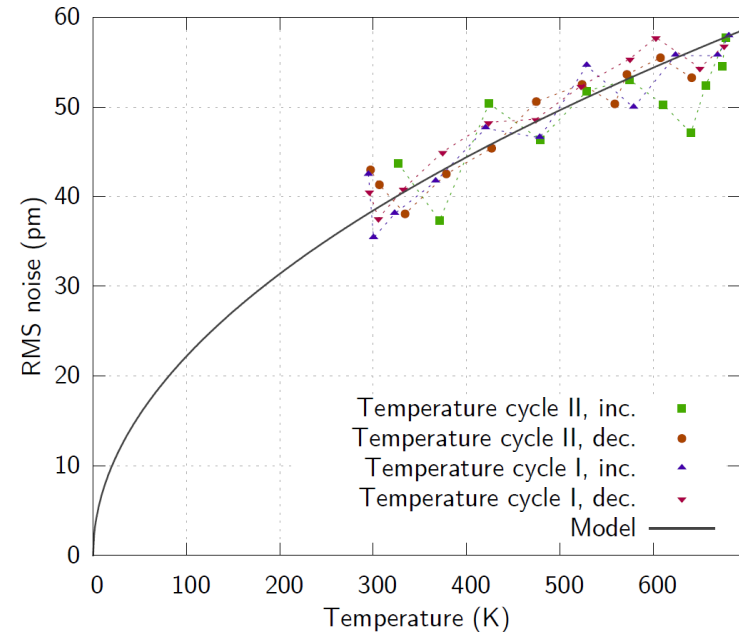
- Second order mechanical system.
- Johnson-Nyquist noise.

$$x_n = \frac{L \sqrt{4k_B T \frac{\omega_0 J}{Q}}}{2J\omega_0 \sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \frac{1}{Q^2} \left(\frac{\omega}{\omega_0}\right)^2}}$$



# Experimental validation

- RMS for a bandwidth of 13 Hz around the resonance frequency.
- The thermomechanical noise increases with temperature.
- Decent values for other fitting parameters (i.e.  $\omega_0$ ,  $J$  and  $Q$ ).



# Case I: MEMS accelerometer resolution

$$\tau_n = \sqrt{J} \cdot \dot{\theta}_n, \quad J = \frac{1}{7} mL^2,$$

with  $m$  the mass of the channel with the fluid, the noise angle density squared  $\langle \theta_n^2 \rangle$  follows from

$$\langle \theta_n^2 \rangle = \frac{1}{2\pi} \int_0^\infty |\theta_n(\omega)|^2 d\omega.$$

And thus, Eq. (9) can be solved.

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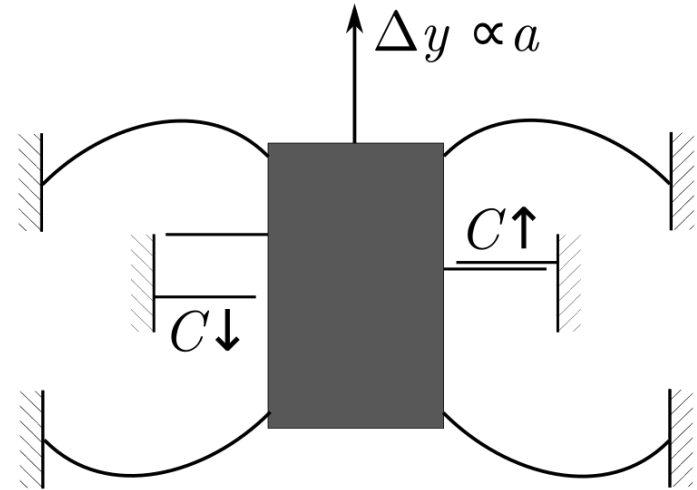
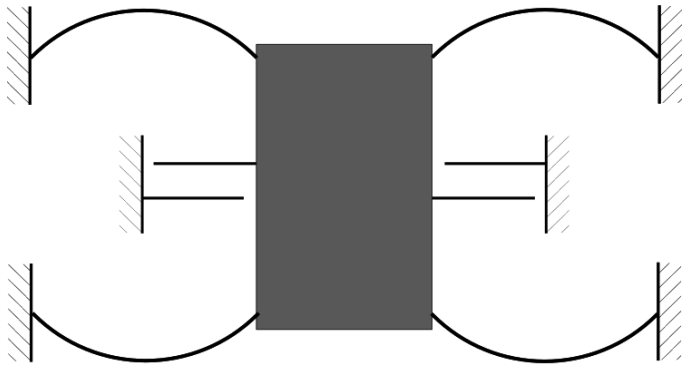
$$|x_n(\omega)| \approx \frac{L}{2} \cdot \frac{\sqrt{4k_B T \frac{\omega_0 Q}{Q}}}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

where the fundamental limit to the minimum flow sensors is given by the noise floor with currently the best resolution is given by van der Meulen et al. [9]. This sensor was a microfluidic channel with a diameter of approximately 14 ng s<sup>-1</sup> was reported. The dominant noise is caused by electronics.

MEMS accelerometers have been studied for microfluidic flow measurement. Atomic force microscopy (AFM) and scanning electron microscopy (SEM) have been used to measure the fundamental noise limit. To validate the result by measuring the noise equivalent mass flow, we will study the thermomechanical noise on the MEMS accelerometers defining the noise equivalent mass flow sensors.

The channel can be modeled as a second order system. The time constant is given by

# Accelerometer



# Accelerometer

- Noise acceleration

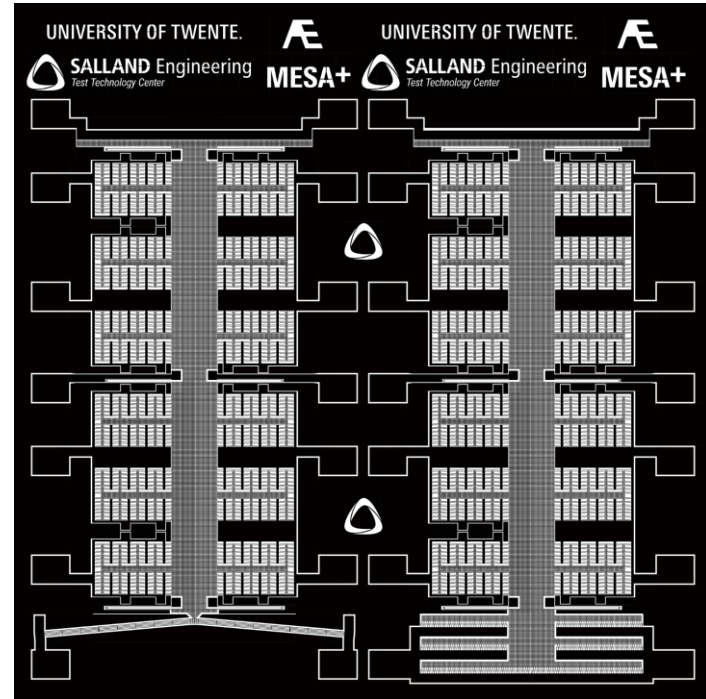
$$a_n = 240 \mu g / \sqrt{\text{Hz}}$$

- Displacement

$$x_n = 5 \text{ pm} / \sqrt{\text{Hz}}$$

- Difference in capacitance

$$C_n = 600 \text{ zF} / \sqrt{\text{Hz}}$$





# Case II: MEMS flow meter resolution

$$\tau_n = \sqrt{J} \cdot \dot{\theta}_n, \quad J = \frac{1}{7} mL^2,$$

with  $m$  the mass of the channel with the fluid, the noise angle density squared  $\langle \theta_n^2 \rangle$  follows from

$$\langle \theta_n^2 \rangle = \frac{1}{2\pi} \int_0^\infty |\theta_n(\omega)|^2 d\omega.$$

And thus, Eq. (9) can be solved.

$$\frac{K}{2\pi} \int_0^\infty |\theta_n(\omega)|^2 d\omega = k_B T.$$

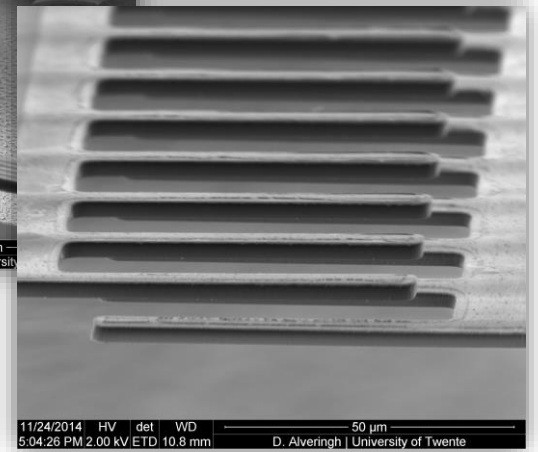
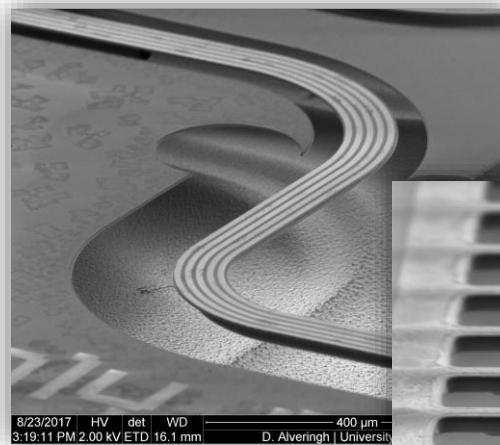
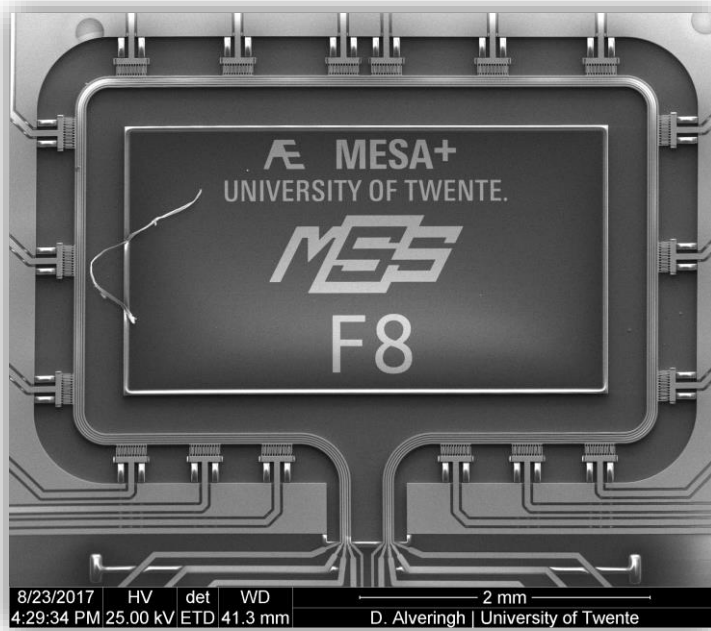
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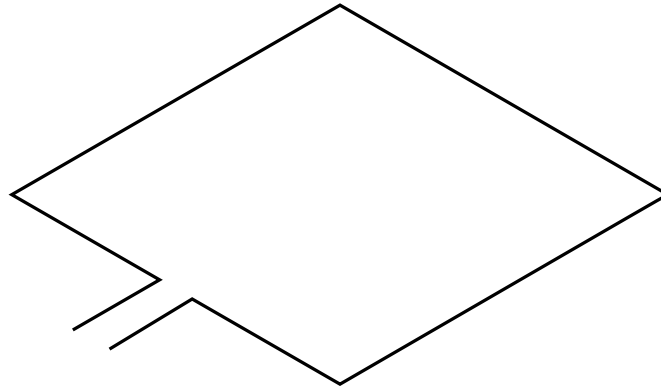
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$$|x_n(\omega)| \approx \frac{L}{2} \cdot \frac{\sqrt{4k_B T \frac{\omega Q}{Q}}}{\sqrt{\frac{1}{2} \left( \frac{1}{\omega} + \frac{1}{\omega} \right) + \frac{1}{\omega^2}}}$$

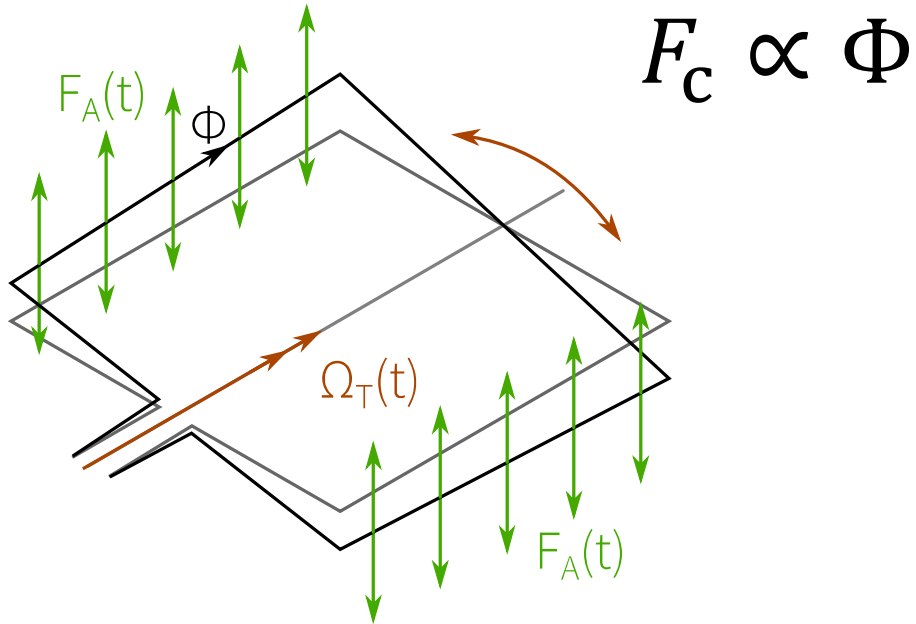
# Microfabricated Coriolis flow sensor



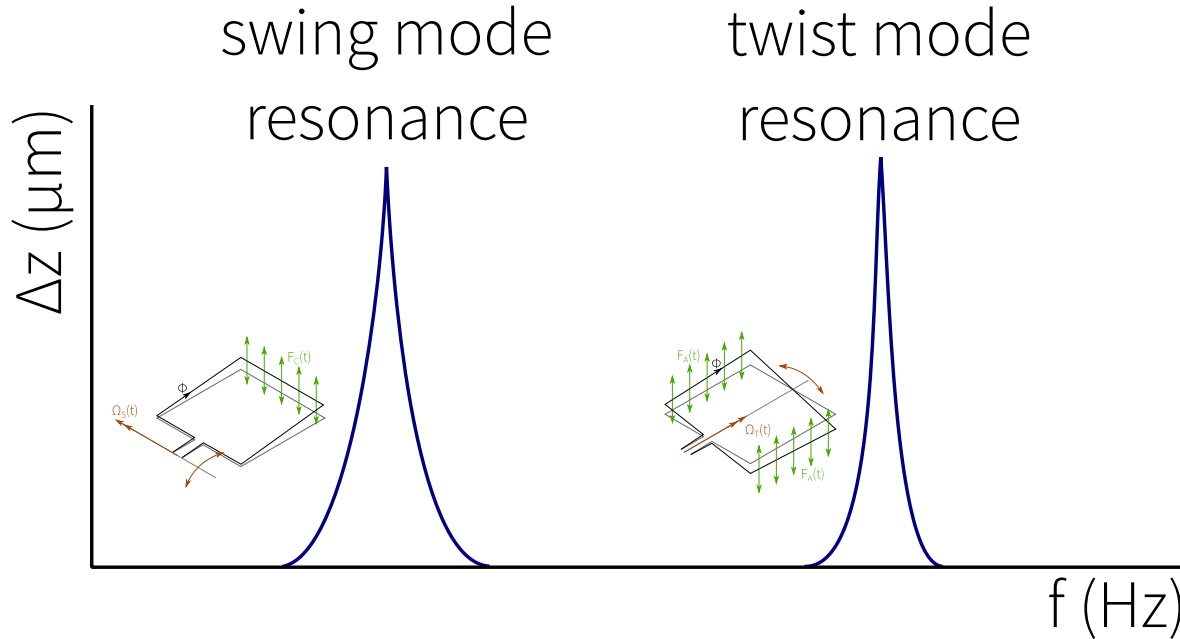
# Microfabricated Coriolis flow sensor



# Microfabricated Coriolis flow sensor



# Microfabricated Coriolis flow sensor



- Twist noise on twist.
- Twist noise on swing.
- Swing noise on swing.
- Swing noise on twist.

# Microfabricated Coriolis flow sensor

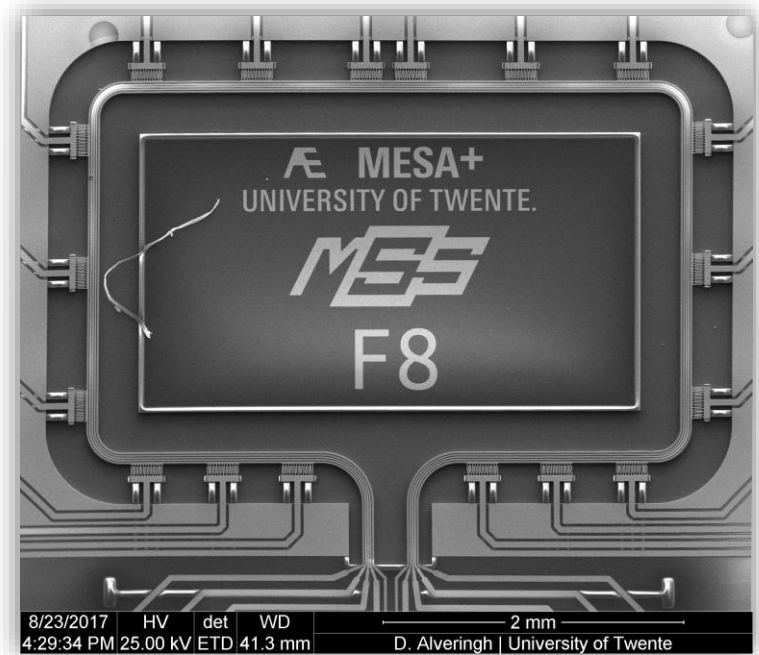
- Limited by noise when  
 $F_c = F_{\text{swingnoise@twistfrequency}}$

- Mass flow

$$\Phi_n = 0.3 \text{ ng s}^{-1} / \sqrt{\text{Hz}}$$

- Difference in phase

$$\phi_n = 3 \text{ n}^\circ / \sqrt{\text{Hz}}$$



# Conclusion

The fundamental limit to the angular flow sensors is given by the thermal noise in the channel with currently the best resolution [9]. This sensor was able to measure a flow rate of approximately  $14 \text{ ng s}^{-1}$  was reported. The dominant noise is caused by the thermal noise in the channel.

Microfluidic sensors have been studied for microfluidic sensors [12], atomic force microscopy [13], and optical beams [17]. However, the noise in microfluidic sensors have never been studied. Here, we study the fundamental noise limits in microfluidic sensors, validate the result by measuring the noise equivalent mass flow rate, and compare the thermomechanical noise on the channel with the noise equivalent mass flow rate of the flow sensors.

The noise equivalent mass flow rate can be modeled as a second order system. The noise equivalent mass flow rate is given by

where  $\tau_n$  can be modeled as a second order system. The noise equivalent mass flow rate is given by

where  $\tau_n$  can be modeled as a second order system. The noise equivalent mass flow rate is given by

$$\tau_n = \sqrt{\frac{4k_B T R}{Q}} \cdot \sqrt{J}, \quad J = \frac{1}{7} m L^2,$$

with  $m$  the mass of the channel with the fluid. The noise angle density squared  $\langle \theta_n^2 \rangle$  follows from

$$\langle \theta_n^2 \rangle = \frac{1}{2\pi} \int_0^\infty |\theta_n(\omega)|^2 d\omega.$$

And thus, Eq. (9) can be solved.

$$\frac{K}{2\pi} \int_0^\infty |\theta_n(\omega)|^2 d\omega = k_B T.$$

Inserting Eq. (12) into Eq. (9) and performing the integration gives the noise torque spectral density  $\tau_n$ :

$$\tau_n = \sqrt{4k_B T R},$$

which can be interpreted as a mechanical Johnson–Nyquist noise. In our experiments, we measured the displacement spectral density  $|x_n(\omega)|$  at the channel inlet and outlet. Substituting Eq. (14) into Eq. (10) and multiplying by  $L/2$  gives an expression for this noise density as a function of frequency:

$$|x_n(\omega)| \approx \frac{L}{2} \cdot \frac{\sqrt{4k_B T \frac{\omega_0 l}{Q}}}{\sqrt{\omega^2 + \omega_0^2}}$$

## Conclusion

- The Johnson-Nyquist noise model is a convenient method for multidomain analog resolution analysis.
- From two types of MEMS sensors, future specifications for capacitance meters and phase detectors are derived.
- These are in the order of aF and n° respectively.

$$J = \frac{1}{7} m L^2 \omega^2$$

with  $m$  the mass of the channel with the fluid, the noise angle density squared ( $\theta_n^2$ ) follows from

$$\langle \theta_n^2 \rangle = \frac{1}{2\pi} \int_0^\infty |\theta_n(\omega)|^2 d\omega.$$

And thus, Eq. (9) can be solved.



$$|\theta_n(\omega)| \approx \frac{L}{2} \frac{\sqrt{4k_B T \frac{\omega d}{Q}}}{\dots}$$



# Outlook

- In practice: sampling rate and high-voltage compatibility are more relevant at the moment.

11:00 - Break with drinks and snacks

14:30 - Break with drinks and snacks

**MEMS TECHNOLOGY**

- + Thermomechanical noise - Dennis Alvering - Salland Engineering
- + PZT Microcantilever Sensors - Aleksandar Andreski - Saxion/Tech4Future
- + Low capacitance - Armando Bonilla Fernandez - Salland Engineering

**MEMS PRODUCTS**

- + CMUT MEMS - Rob van Schaijk - Philips Innovation Services
- + Automotive MEMS pressure Sensors - Gerard Klaasse - Sensata

13:00 - Lunch buffet

14:30 - Presentation Solidus MEMS Tester

15:30 - Social Event - drinks and snacks


# Acknowledgements



**UNIVERSITY OF TWENTE.**



 D. Alveringh, *Integrated throughflow mechanical microfluidic sensors*, PhD dissertation, 2018.

 D. Alveringh, *Experimental analysis of thermomechanical noise in micro Coriolis mass flow sensors*, *Sensors and Actuators A: Physical* 271: 212-216, 2018.