Thermomechanical noise in micromachined sensors

The fundamental specification for MEMS test equipment?

Dennis Alveringh


clap
t
f
low voice
noise
t
f
f
high voice
t
f

SALLAND Engineering
Test Technology Center

Outline

- ~~Introduction~~
- Noise theory
- Experimental validation
- Case I: MEMS accelerometer resolution
- Case II: MEMS flow meter resolution
- Conclusion

Noise theory

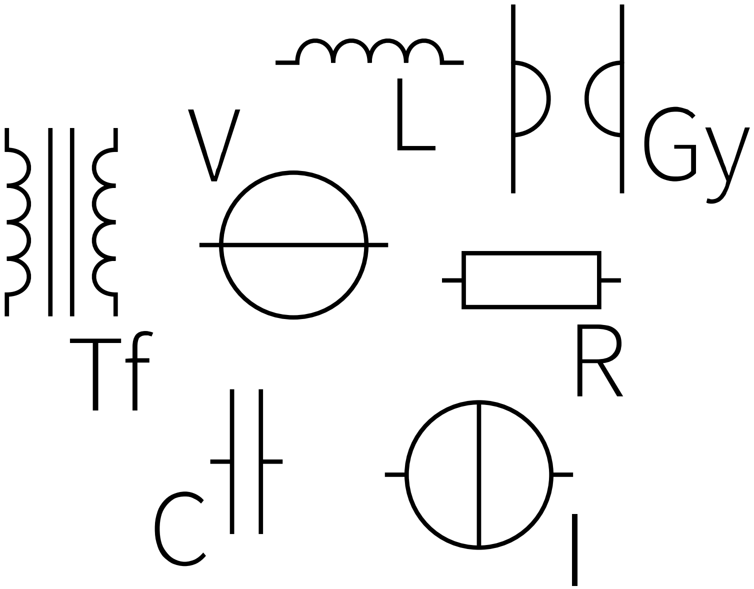
SALLAND Engineering
Test Technology Center

Modeling noise

- Thermomechanical noise.
- Johnson-Nyquist noise.
- Electrical engineers

♥

lumped element
modeling!



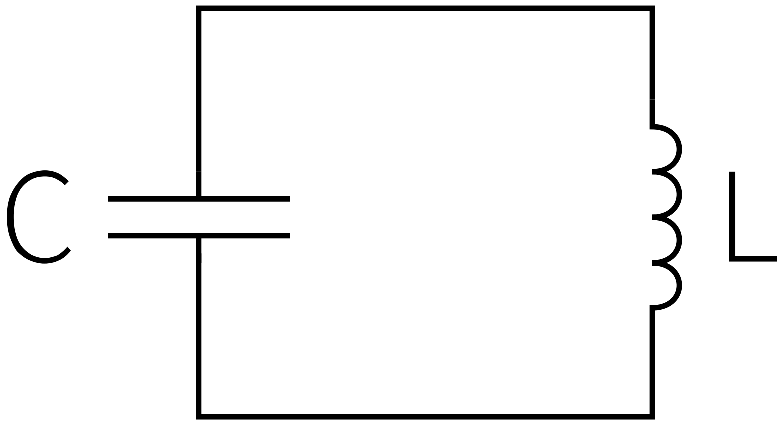
A simple system

- 'Kinetic' energy (inductor)

$$E_{\mathrm{k}} = \frac{1}{2} L i^2$$

- 'Potential' energy (capacitor)

$$E_{\mathrm{p}} = \frac{1}{2} C v^2$$



SALLAND Engineering
Test Technology Center

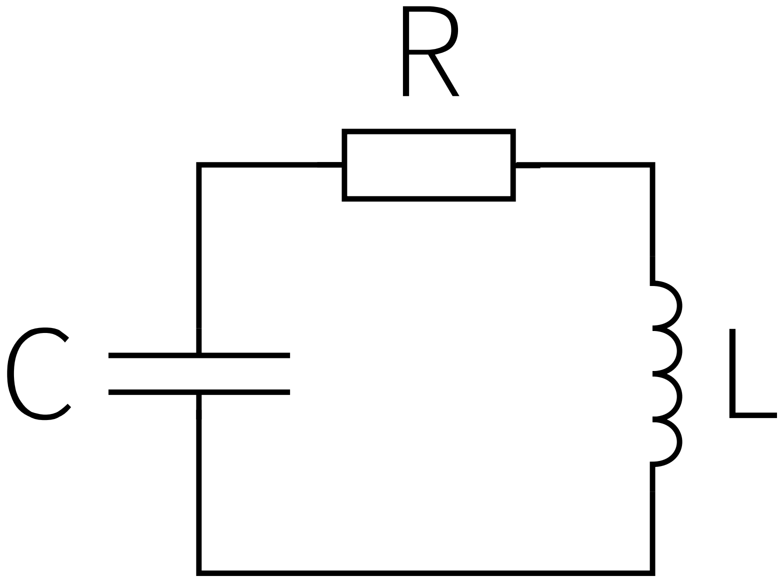
A simple system, but WRONG!

- ‘Lost’ energy in the resistor

$$P_R = \frac{E}{t} = i^2 R$$

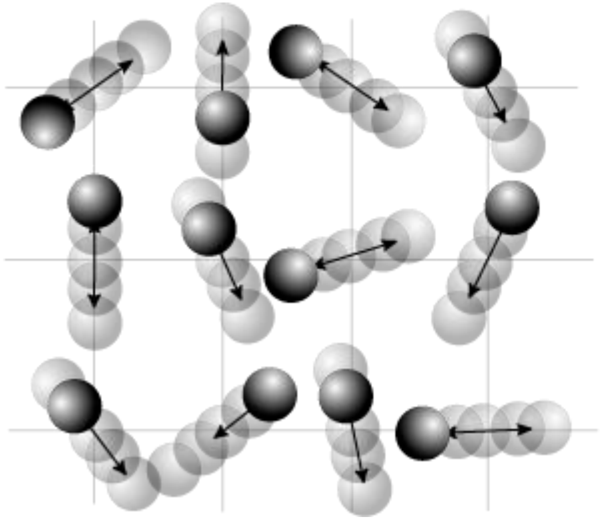
- Ideal electrical system.
  Infinitesimal heat capacity.

- Conservation of energy?



Equipartition theorem

$$\sum_{\text{storage}} \langle E \rangle = k_{\mathrm{B}} T$$



📖 D. Alveringh, *Experimental analysis of thermomechanical noise in micro Coriolis mass flow sensors,* Sensors and Actuators A: Physical 271: 212-216, 2018.

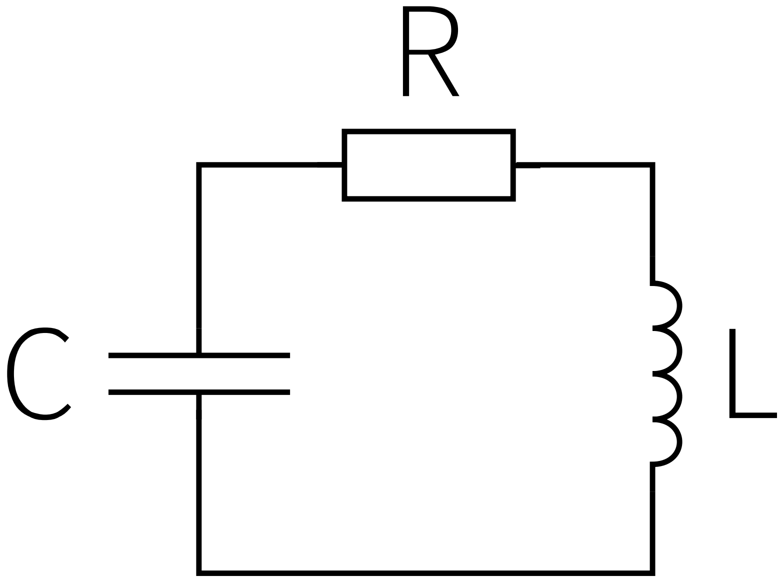
A simple system

- Equipartition theorem

$$\sum_{\text{storage}} \langle E \rangle = k_{\mathrm{B}} T$$

- Energy storage in this system

$$\frac{1}{2} L \langle i_n^2 \rangle + \frac{1}{2} C \langle v_n^2 \rangle = k_{\mathrm{B}} T$$



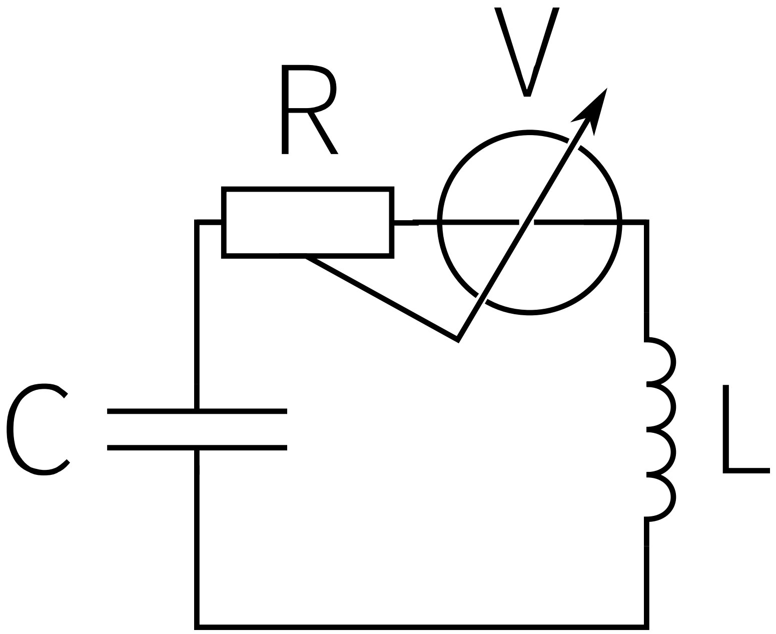
A simple system

- Equipartition theorem

$$\sum_{\text{storage}} \langle E \rangle = k_{\mathrm{B}} T$$

- Energy storage in this system

$$\frac{1}{2} L \langle i_{\mathrm{n}}^2 \rangle + \frac{1}{2} C \langle v_{\mathrm{n}}^2 \rangle = k_{\mathrm{B}} T$$



SALLAND Engineering
Test Technology Center

A simple system

- Apply some Kirchhoff and Fourier...

$$i_{\mathrm{n}}(\omega) = \frac{v_{\mathrm{n}}(\omega)}{j\omega RC - \omega^2 LC + 1}$$

- Calculate the average...

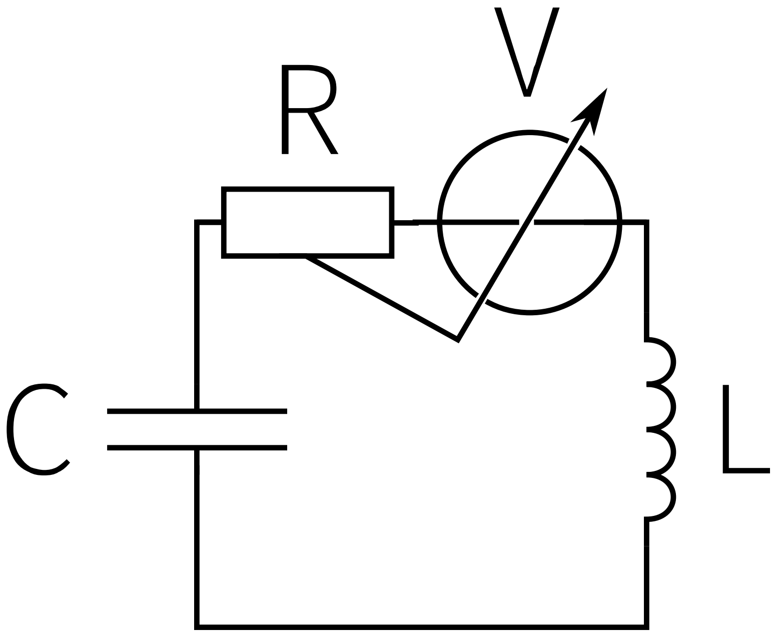
$$\langle i_{\mathrm{n}}^2 \rangle = \frac{1}{2\pi} \int_0^\infty i_{\mathrm{n}}(\omega)^2 \, d\omega$$

- Apply equipartition theorem...

$$L\langle i_{\mathrm{n}}^2 \rangle = k_{\mathrm{B}} T$$

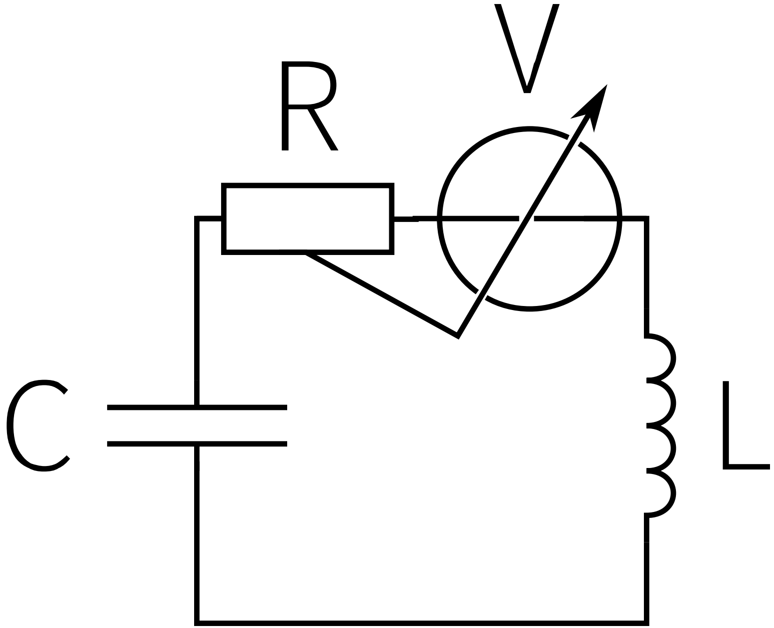
- Result....

$$v_{\mathrm{n}}^2 = 4 k_{\mathrm{B}} T R$$



Johnson–Nyquist noise

$$v_{\mathrm{n}} = \sqrt{4k_BTR}$$



SALLAND Engineering
Test Technology Center

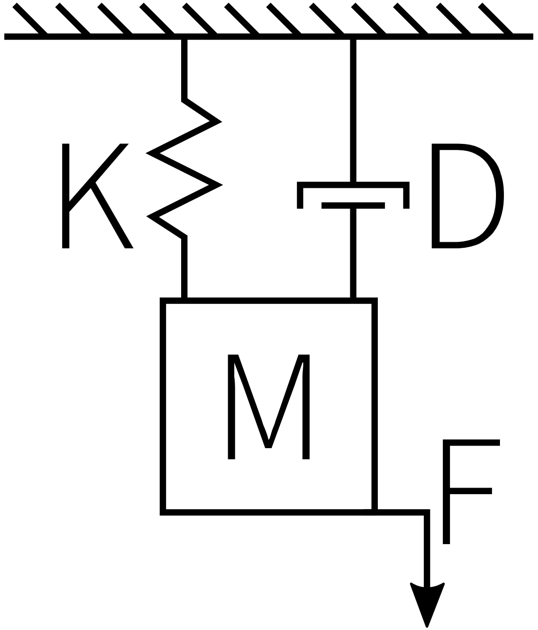
Johnson–Nyquist noise

- ‘Kinetic’ energy (mass)

$$E_{\mathrm{k}} = \frac{1}{2} M u^2$$

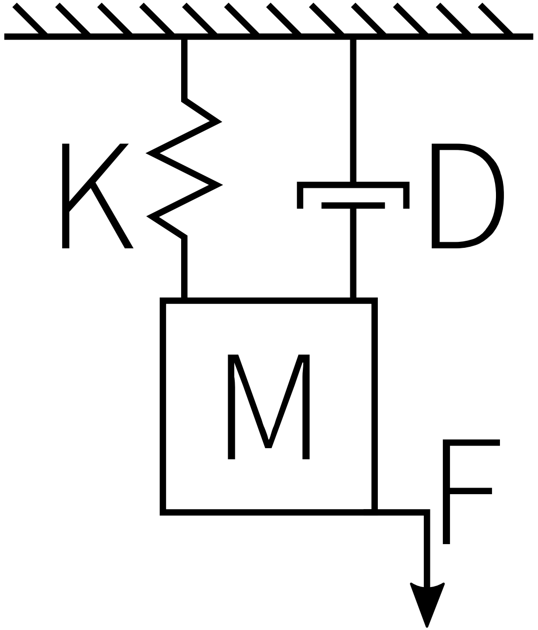
- ‘Potential’ energy (spring)

$$E_{\mathrm{p}} = \frac{1}{2} K x^2$$



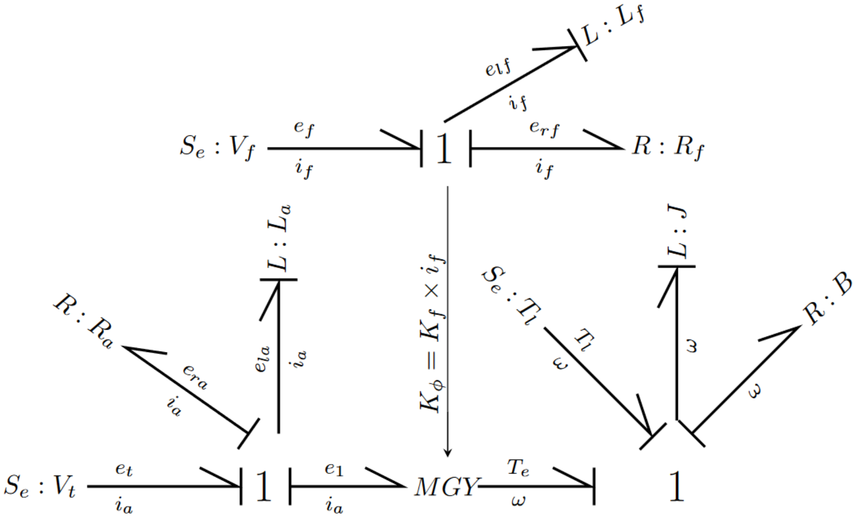
Johnson–Nyquist noise

$$F_{\mathrm{n}} = \sqrt{4k_B T D}$$



Some practical tips on this trick...

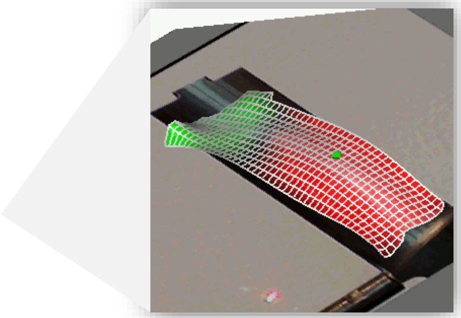
- Take care of multiple degrees of freedom.
- Think outside the boundaries of one domain.
- Make sure all ‘energy storages’ of the system are taken into account.

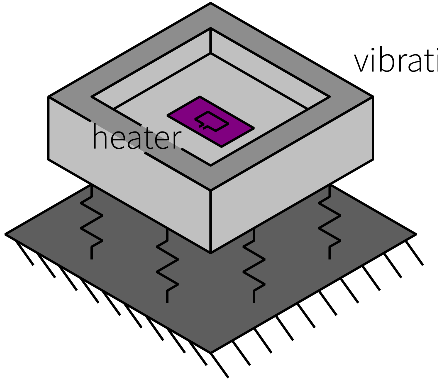


SALLAND Engineering
Test Technology Center

Experimental validation

Measurement setup

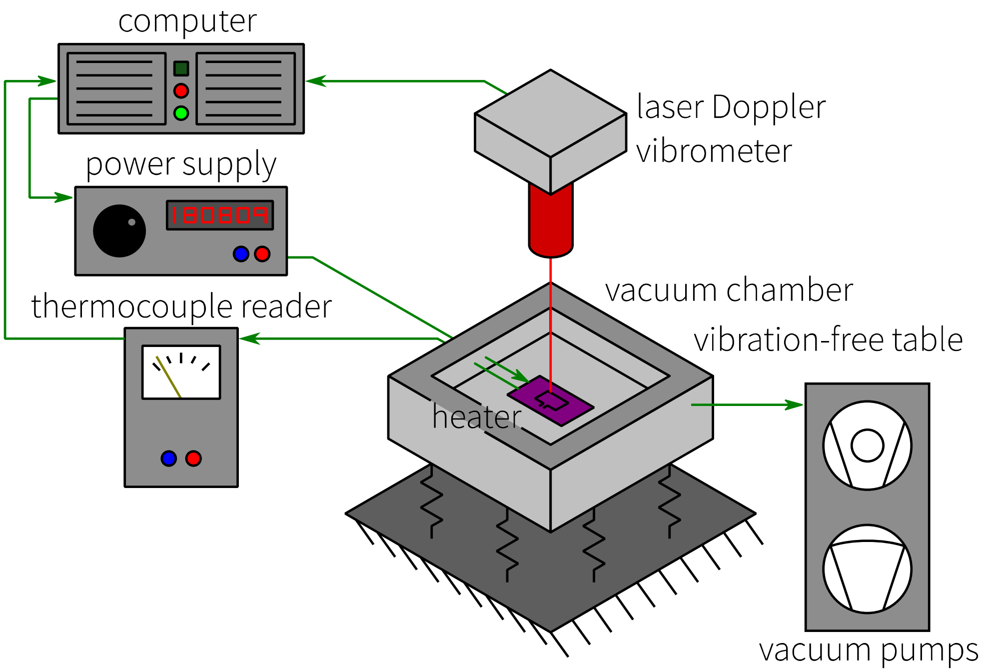




📖 D. Alveringh, *Integrated throughflow mechanical microfluidic sensors,* PhD dissertation, 2018.

SALLAND Engineering
Test Technology Center

Measurement setup

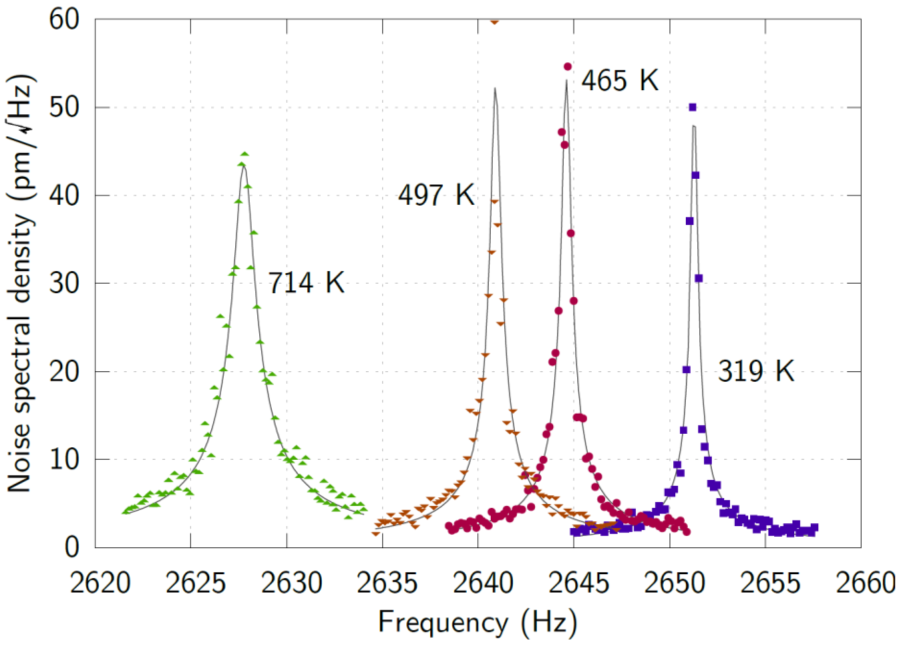


📖 D. Alveringh, *Integrated throughflow mechanical microfluidic sensors,* PhD dissertation, 2018.

Model fits

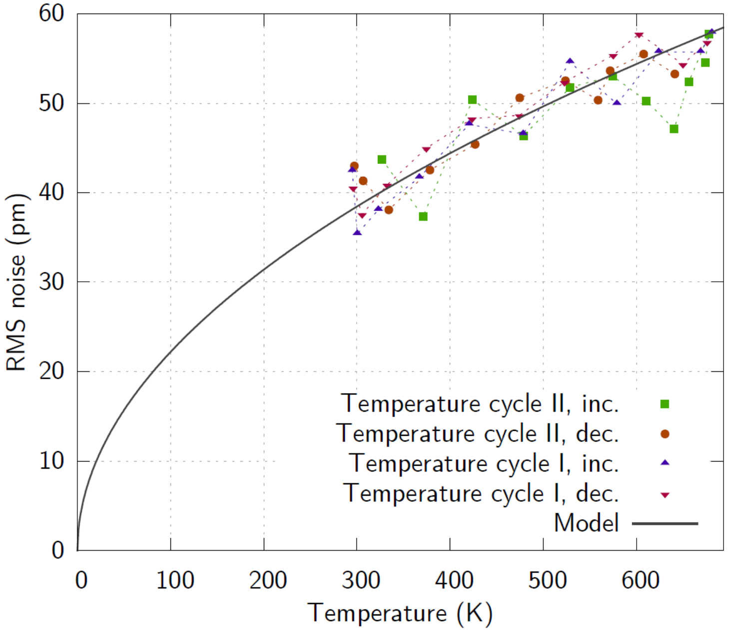
- Second order mechanical system.
- Johnson-Nyquist noise.

$$x_n = \frac{L\sqrt{4k_B T \frac{\omega_0 J}{Q}}}{2J\omega_0\sqrt{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2+\frac{1}{Q^2}\left(\frac{\omega}{\omega_0}\right)^2}}$$



Experimental validation

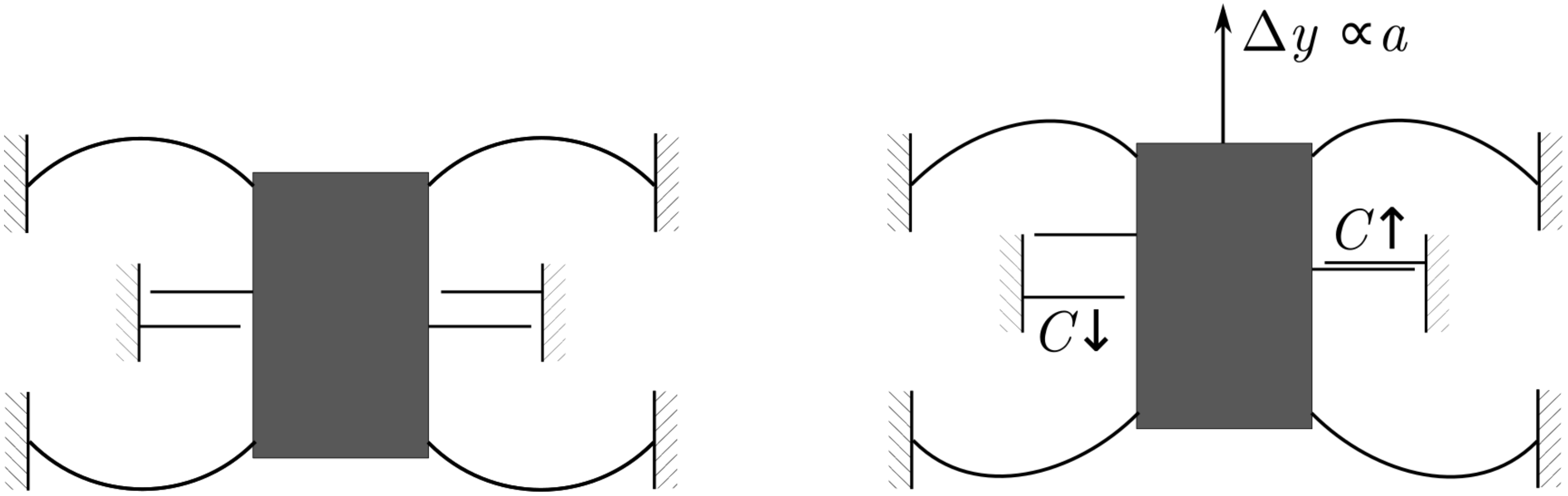
- RMS for a bandwidth of 13 Hz around the resonance frequency.
- The thermomechanical noise increases with temperature.
- Decent values for other fitting parameters (i.e. ω_0, J and Q).



Case I: MEMS accelerometer resolution

SALLAND Engineering
Test Technology Center

Accelerometer



SALLAND Engineering
Test Technology Center

Accelerometer

- Noise acceleration

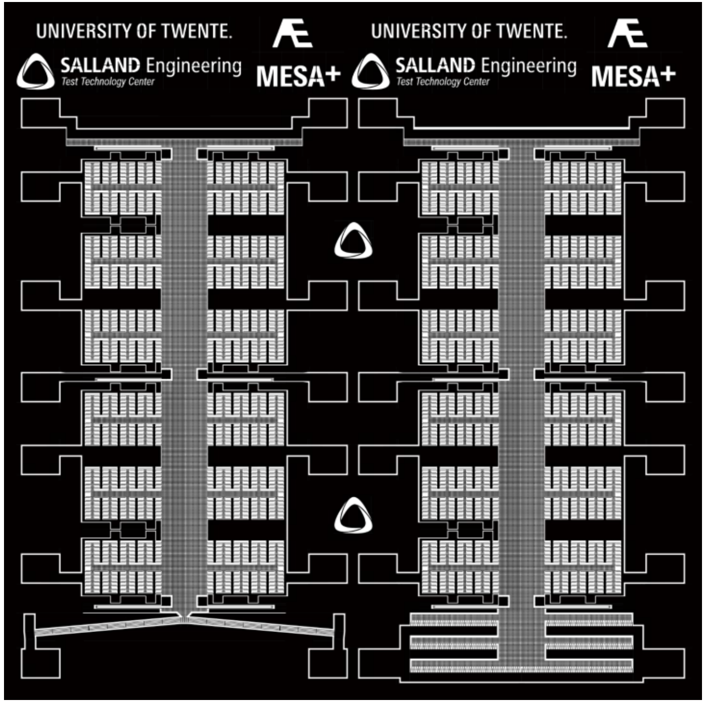
$$a_{\mathrm{n}} = 240\ \mu g/\sqrt{\mathrm{Hz}}$$

- Displacement

$$x_{\mathrm{n}} = 5\ \mathrm{pm}/\sqrt{\mathrm{Hz}}$$

- Difference in capacitance

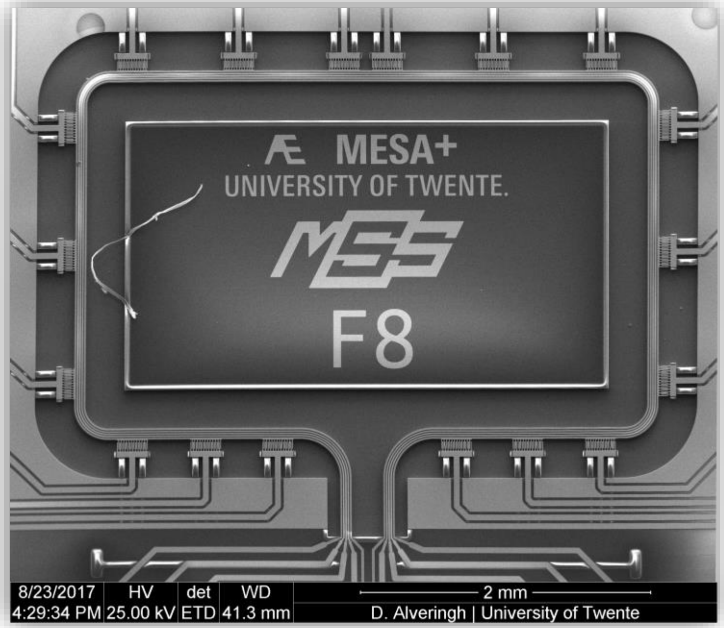
$$C_{\mathrm{n}} = 600\ \mathrm{zF}/\sqrt{\mathrm{Hz}}$$

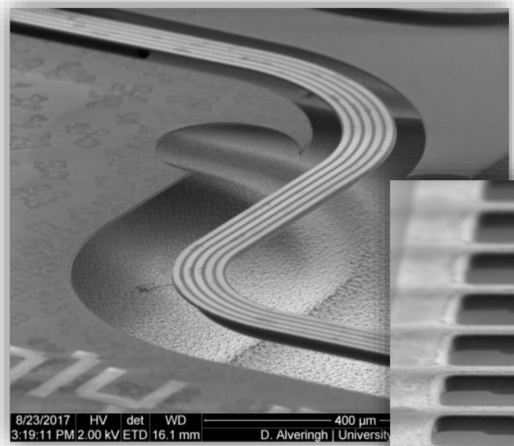


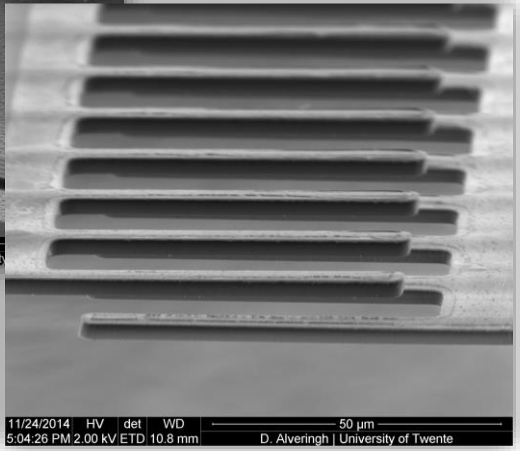
SALLAND Engineering
Test Technology Center

Case II: MEMS flow meter resolution

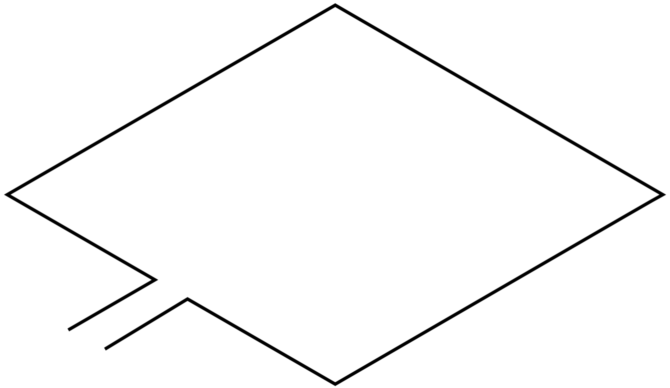
Microfabricated Coriolis flow sensor



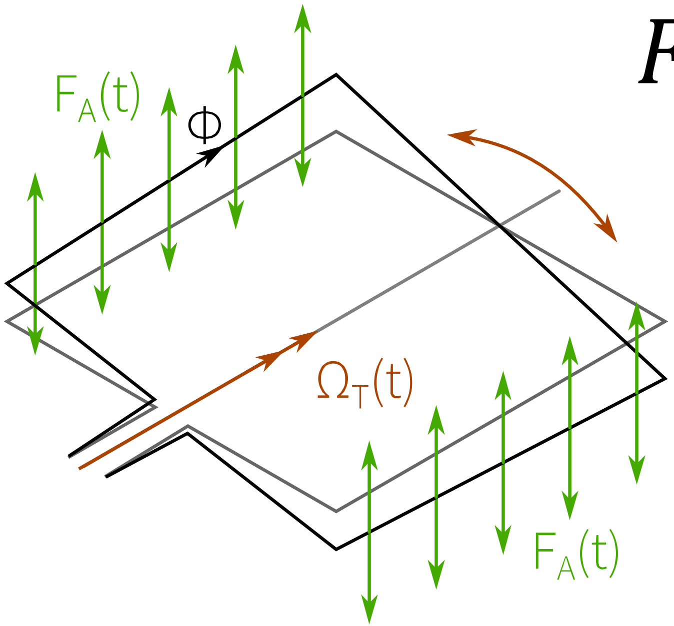




Microfabricated Coriolis flow sensor

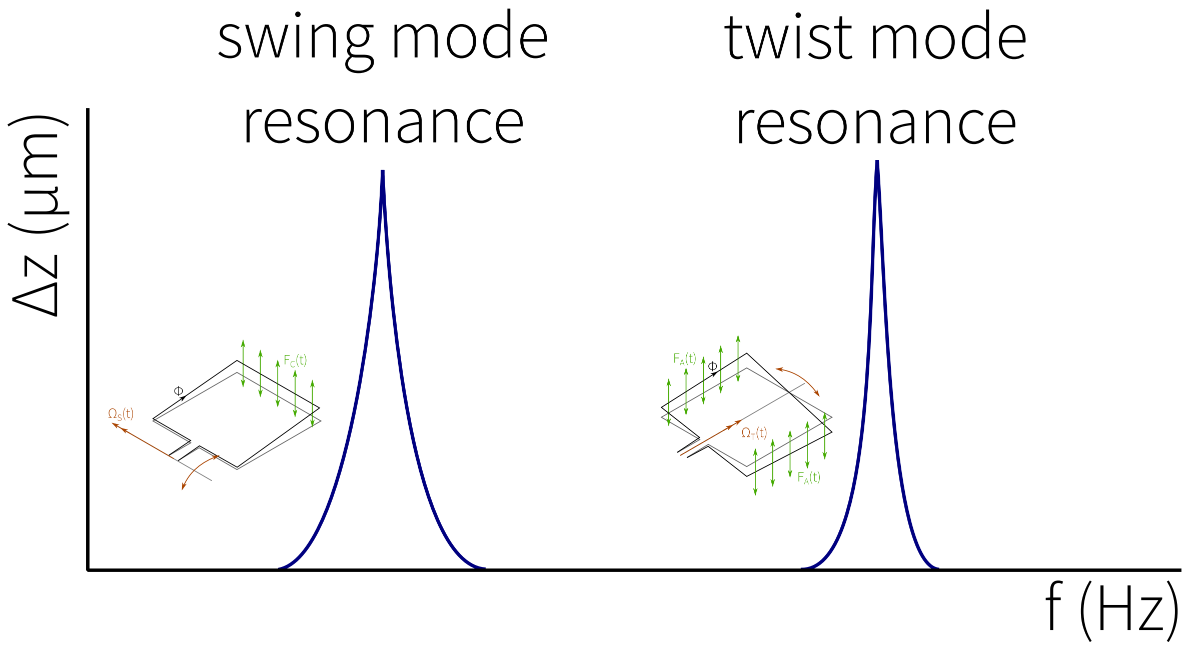


Microfabricated Coriolis flow sensor



$$F_c \propto \Phi$$

Microfabricated Coriolis flow sensor



- Twist noise on twist.
- Twist noise on swing.
- Swing noise on swing.
- Swing noise on twist.

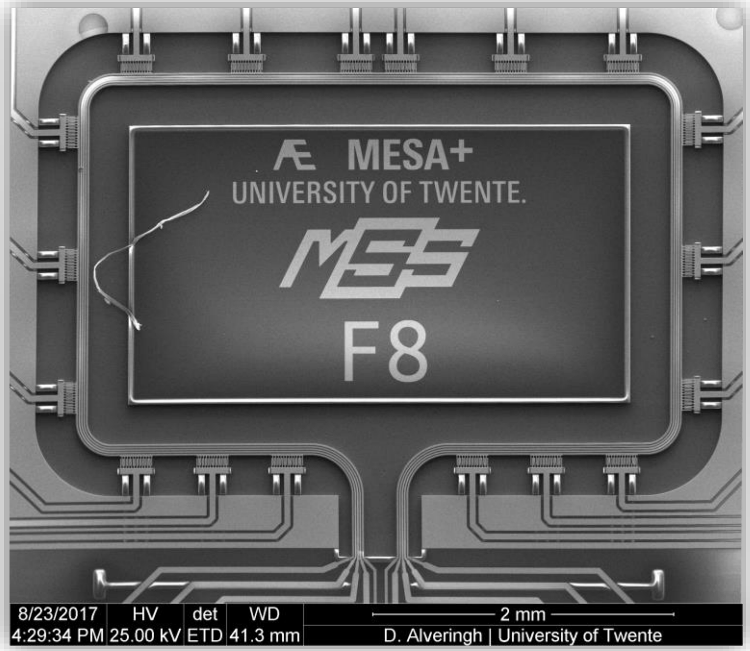
SALLAND Engineering
Test Technology Center

Microfabricated Coriolis flow sensor

- Limited by noise when
  $F_c = F_{\mathrm{swingnoise@twistfrequency}}$

- Mass flow
  $$\Phi_n = 0.3\ \mathrm{ng\ s^{-1}}/\sqrt{\mathrm{Hz}}$$

- Difference in phase
  $$\phi_n = 3\ \mathrm{n°}/\sqrt{\mathrm{Hz}}$$



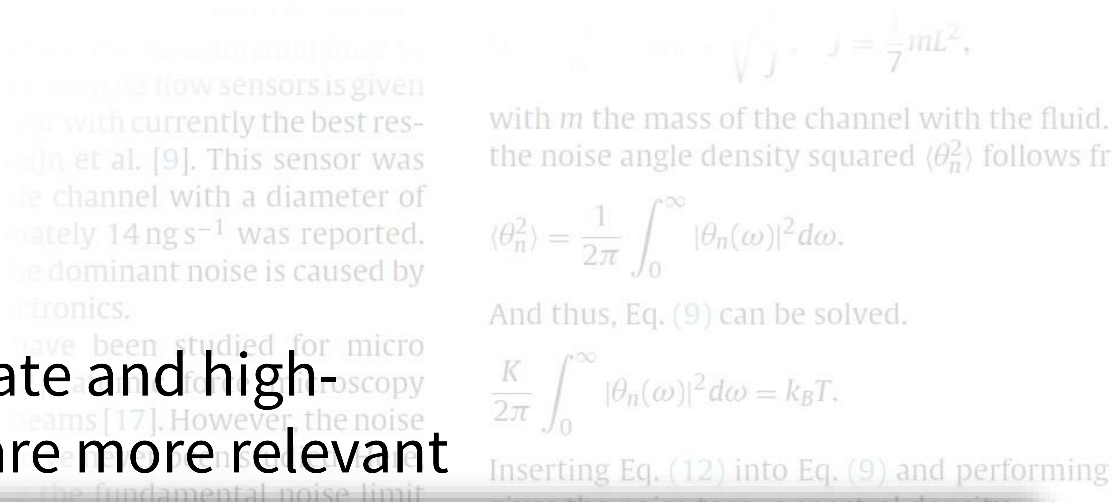
Conclusion

Conclusion

- The Johnson-Nyquist noise model is a convenient method for multidomain analog resolution analysis.
- From two types of MEMS sensors, future specifications for capacitance meters and phase detectors are derived.
- These are in the order of aF and n° respectively.

Outlook

- In practice: sampling rate and high-voltage compatibility are more relevant at the moment.


with m the mass of the channel with the fluid.
the noise angle density squared $\langle\theta_n^2\rangle$ follows fr

$$\langle\theta_n^2\rangle = \frac{1}{2\pi}\int_0^\infty |\theta_n(\omega)|^2 d\omega.$$

And thus, Eq. (9) can be solved.

$$\frac{K}{2\pi}\int_0^\infty |\theta_n(\omega)|^2 d\omega = k_B T.$$

Inserting Eq. (12) into Eq. (9) and performing

Acknowledgements

UNIVERSITY OF TWENTE.

📖 D. Alveringh, *Integrated throughflow mechanical microfluidic sensors*, PhD dissertation, 2018.

📖 D. Alveringh, *Experimental analysis of thermomechanical noise in micro Coriolis mass flow sensors*, Sensors and Actuators A: Physical 271: 212-216, 2018.